

# Vibration and Sound

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III and IV, Hyperbolic Tangent of Complex Quantity.  
V, VI and VII, Bessel Functions.  
VIII, Impedance Functions for Piston.  
IX, Legendre Functions.  
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### **Preface to the ASA Edition**

The first edition of *Vibration and Sound* appeared in the mid-30's. It appeared because I could find no text, to use in my class in acoustics, that combined the classical viewpoint of Rayleigh with the emerging perspective engendered by quantum mechanics and the vacuum tube. I learned by classical acoustics under Dayton Miller, when the ear was the ultimate measuring instrument and the equipment was all mechanical. When I came to MIT and took on the responsibilities of teaching the senior acoustics course, I had already become familiar with the analytic powers of wave mechanics and I began to learn the experimental potentialities of electronic equipment. Spurred by my enthusiasm for the new vistas, by the warm support of Professor Richard Fay, and by the continued interest of my students, I spent less than a year in writing the book. I am occasionally told that it helped make MIT an acoustics research center during and after World War II.

*Vibration and Sound* continued to be used, fairly widely, as a text for some two dozen years, going through one revision. When it came, in the 60's to make another revision, my interests had changed and it appeared that the textual popularity of the book had waned. Consequently Ingard and I wrote *Theoretical Acoustics*, incorporating much of *Vibration and Sound* but extending the material enough to make it a monograph rather than an intermediate text. Evidently the subsequent disappearance of *Vibration and Sound* was premature for, during the 70's, I received many pleas for another printing. Therefore I welcome the offer of the Acoustical Society of America to make the text available once more, and I hope that the response from the acoustical public will justify their decision.

Philip M. Morse  
Winchester, Mass. June 1981

### **Preface to the Second Edition**

The recent war induced a considerable development in the science of acoustics and in the mathematical techniques that are particularly useful in theoretical acoustics. Studies in ultrasonics have quickened interest in problems of radiation and scattering and in transient phenomena; and the rapid development of microwave techniques, which parallel acoustic techniques, has stimulated interest in the general theory of wave motion.

Reflecting these developments, the present edition of this volume includes more detail than the first edition on radiation problems and introduces the important subject of transient phenomena and the technique of the operational calculus. Both of these subjects are usually shunned because of their reputed difficulty. The writer is convinced that they are not particularly difficult conceptually, although they usually necessitate calculations of rather wearisome extent. They are here discussed with the intent to bring out fundamental ideas, rather than to ensure mathematical rigor of treatment.

In keeping with the plan of the first edition, the more difficult subjects are segregated at the terminations of each chapter, so that they need not be assigned if the book is to be used for a beginning course.

The writer is pleased to acknowledge his indebtedness to a large number of friends for valuable suggestions as to improvements in exposition grammar, arithmetic, and algebra. Some of the most obvious errors in the first edition have been corrected.

Especial thanks are owing J. R. Pellam, who has expended much time and effort in checking the manuscript and mathematics, and to Dr. Cyril Harris for his many helpful suggestions concerning subject matter.

Philip M. Morse  
Upton, N.Y., January, 1948

### **Preface to the First Edition**

The following book on the theory of vibrations and sound is intended primarily as a textbook for students of physics and of communications engineering. After teaching the introductory course in this subject at the Massachusetts Institute of Technology for several years, the author has become persuaded that there is need for a new textbook in the field.

There are, of course, many other books on the theory of sound. The author's excuse for adding another to the list is that in the past ten years the rapid growth of atomic physics has induced a complete reorganization of the science of acoustics. The vacuum tube and the other applications of electronics have provided immensely powerful tools for the measurement, recording, and reproduction of sound; tools which have revolutionized acoustic technique. Another useful tool, perhaps not so obvious, is the new mathematical technique which has been developed for the working out of quantum mechanics, and which is capable of throwing light on all problems of wave theory. The last chapter of this book is an example of the utility of these methods. In it the mathematical methods developed for the study of the radiation of light from an atom are applied to the theory of the acoustic properties of rooms.

During the recent rapid change in the science of sound, certain parts of the subject have gained and other parts have lost importance. The present book attempts to follow this change in emphasis and to discuss the new development as well as those portions of the older theory which are still important.

The book has been planned as a textbook with a twofold aim in view. The first aim, of course, is to give the student a general introduction to the theory of vibration and sound. An introductory course in this subject must of necessity be more theoretical than practical. In no other branch of physics are the fundamental measurements so hard to perform, and the theory relatively so simple; and in few other branches are the experimental methods so dependent on a thorough knowledge of theory. Since this is so, the student must first be given a physical picture of the fundamental theory of the vibration of solid bodies and the propagation of sound waves before he can appreciate the techniques used in the measurements of sound, and before he can begin to design acoustical apparatus.

The second aim is to give the student a series of examples of the method of theoretical physics; the way a theoretical physicist attacks a problem and how he finds its solution. This subject is too often neglected, especially in engineering courses. The student is usually given a series of formulas to use in standardized cases, the formulas sometimes introduced by a cursory derivation and sometimes with no derivation at all. After such a course the student is capable of using the formulas on standard problems, but he is unable to devise a new formula to use in unusual cases.

In this book the author has tried to derive every formula from the fundamental laws of physics (there are a few exceptions to this procedure) and to show in some detail the steps in these derivations and their

logical necessity. This does not mean that the mathematical machinery is given in excessive detail, but that the steps in the physical reasoning are brought out. Often generality and mathematical rigor have been sacrificed to make the chain of logic more distinct. It has been the author's experience that once the student can grasp the physical picture behind a mathematical derivation, he can himself add what extra generality and rigor he may need. Often, too, the author has supplemented or replaced the rigid and esoteric technical vocabulary by more colloquial phrases, in order to make vivid a concept, or to suggest a new point of view.

It is assumed that the student has a thorough knowledge of calculus, and some acquaintance with the fundamental laws of mechanics. A knowledge of differential equations is helpful but is not necessary, for the solutions of the various differential equations encountered are worked out in the text. Tables of the functions used are given in the back of the book.

Although the book is designed primarily as a textbook, a certain amount of material of an advanced nature has been introduced. In this way, it is hoped, the volume will be useful as a fairly complete reference work for those parts of the theory of sound which seem at present to be most important for the acoustical scientist. The advanced material has been included in the form of extra sections placed at the end of various chapters. The instructor may assign the first few sections of these chapters for the introductory course, and the student may refer to the other sections for further details when he needs them.

The author wishes to express his gratitude to Professor R.D. Fay and to Dr. W. M. Hall, whose help in choosing subject matter and methods of presentation has been invaluable. He is also indebted to Dr. J.B. Fisk for his willing and painstaking aid in correcting proof, and to many other colleagues in the Department of Physics at the Massachusetts Institute of Technology, for their many helpful suggestions.

Philip M. Morse  
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