Vibration of Shells

Arthur W. Leissa

Published in 1993; Originally Published in 1973

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Preface to the Reprinted Edition

The Acoustical Society of America is reprinting this book, along with my previously published one, "Vibration of Plates," as a service to the technical community with no commercial expectations. I am very pleased that they are doing this, for both books have been in great demand, and have been out of print for a long time.

I wrote the two books because I felt that there was a great need for them. At least I needed them, and therefore thought that many others did, too. In this respect my expectations have been exceeded. During the intervening two decades since they appeared, I have received scores of letters and phone calls from people, often asking questions, who are grateful for their existence. *Science Citation Index* shows that both books are cited many times every year in the published literature. Thus, I feel that my time was well spent in writing them.

"Vibration of Plates" refers to approximately 500 research publications, whereas "Vibration of Shells" includes approximately 1000. This is not surprising, because the scale of knowledge for shells is at least one order of magnitude greater than for plates. Most obviously, whereas a plate is *one* special case of a shell (zero curvature), there are innumerable other shells having their distinct curvatures (e.g., circular cylindrical, noncircular cylindrical, conical, spherical, ellipsoidal, paraboloidal, toroidal, hyperbolic paraboloidal). Moreover, the "complicating effects" such as anisotropy, initial stresses, variable thickness, surrounding media (e.g., water, air), large (nonlinear) deflections, shear deformation and rotary inertia, and nonhomogeneity (including laminated composites) which may be present in plates are all equally possible for shells. But, the number of possible types of "simple" boundary conditions (i.e., not including elastic supports) is also far greater for a shell than for a plate. Whereas any of 4 sets of boundary conditions exist mathematically for a typical edge of a plate (although one is physically impossible), 16 sets exist for a shell boundary.

Not only are there more problems to be studied for shells than for plates, but the *need* for their study is greater. Shells are the most efficient structures available to mankind, and they should be used more in design than they are. They are certainly more difficult to design and fabricate than plates. Fabrication difficulties are being decreased as, for example, with laminated composite materials where curved surfaces are equally easy to lay up as are flat surfaces. Design is held back by the complexity of shell analysis and the need for more complete information in design handbooks or in summarizing monographs such as this one.

In spite of this great need, much less has been published about shell vibrations than about plate vibrations during the past two decades. Whereas the plate vibration literature has at least quadrupled (to more than 2000 references), the rate of publication in shell vibrations has probably *decreased*. Although I cannot document this statement with numbers, I believe that the numbers of shell vibration papers during each of the past two decades are less than that for the preceding decade (1963-1973). This is unfortunate. Whereas many analysts capable of dealing with these complicated problems have moved on to other challenging problems in the field of mechanics, not many have taken their places.

A great deal of shell analysis, including vibration analysis, is currently being done with finite element computer programs. Such computational capability is marvelous when the analyst formulating the computer program understands shell theory and applications, and the user (e.g., designer) understands how shells can behave. Otherwise very poor results may be obtained. I often see such poor results in the published literature; the authors, who are supposedly knowledgeable researchers, are not even aware that they are poor because they have not bothered to check their analytical procedure against accurate results which are available in certain special cases.

Readers interested in further general perspectives on shell vibrations are invited to peruse the paper "The Relative Complexities of Plate and Shell Vibrations" which I wrote for *The Shock and Vibration Bulletin* (Bulletin 50, Part 3, pp. 1-9, Sept. 1980).

As could be expected, some errors did appear in this work. A total of seven errors were found that have been corrected in this reprinted edition. Corrections are on pages 32, 35, 81, 125, 132, 309, and 321.

I want to thank Dr. Mauro Pierucci of San Diego State University who, as Chairman of the Book Committee of the Acoustical Society of America, initiated this reprinting effort and followed it through to its fruition.

Arthur W. Leissa
The Ohio State University
May 10, 1993

Preface to the Original Edition

This monograph is the second in a series dedicated to the organization and summarization of knowledge existing in the field of continuum vibrations. The first monograph, entitled *Vibration of Plates*, was published in 1969, also by the National Aeronautics and Space Administration.

The objectives of the present work are the same as those of the previous one, namely, to provide

(1) A comprehensive presentation of available results for free vibration frequencies and mode shapes which can be used by the design or development engineer.

(2) A summary of known results for the researcher, facilitating comparison of future theoretical and experimental results, and delineating by implication those problems which need further study.

The scope of the present monograph is also the same as that of the previous one in that

(1) Materials are assumed to be linearly elastic.

(2) Structures were not included in this study, although some attention has been given to the accuracy of representing a stiffened shell as an orthotropic shell for purposes of vibration analysis.

The key to a comprehensive monograph such as this is organization. Careful organization not only makes the completed work more understandable and useful to the reader, but also facilitates the writing. Although much of the organization can be seen from the Contents, I will attempt to explain it further below.

Shells have all the characteristics of plates along with an additional one--curvature. This we have cylindrical (noncircular, as well as circular), conical, spherical, ellipsoidal, paraboloidal, toroidal, and hyperbolic paraboliodal shells as practical examples of various curvatures. The plate, on the other hand, is the special limiting case of a shell having no curvature. So called "curved plates" found in the literature are, in reality, shells. Thus, the primary classifier of the field of shell vibrations is chosen to be curvature. For a given curvature (say circular cylindrical, for example) the available literature is dived as to whether complicating effects such as anisotropy, initial stresses, variable thickness, large deflections, nonhomogeneity, shear deformation and rotary inertia, and the effects of surrounding media are present or not. The next subdivision of organization is boundary shape. Thus, a circular cylindrical shell can be open or closed, have boundaries which are parallel to the principal coordinates or not, and have cutouts or not. Once the boundary shape is determined, attention is given to the possible types of fixity that can exist along each edge (i.e., the boundary conditions).. Finally, attention is given to such special considerations as point supports or added point masses. Thus, for each type of curvature, the organization of the previous monograph *Vibration of Plates* is followed.

In addition to having the added complexity of curvature, shells are more complicated than plates because their bending cannot, in general, be separated from their stretching. Thus, a "classical" bending theory of shells is governing by an eighth order system of governing partial differential equations of motion, while a corresponding plate bending theory is only of the fourth order. This added complexity enters into the problem not only by means of more complex equations of motion, but through the boundary conditions as well. The classical bending theory of plates requires only two conditions to be specified along an edge, while a corresponding shell theory requires four specified conditions.

To demonstrate the significance of the latter point, consider a flat panel (i.e., a plate) which is simply supported along two of its opposite edges. The number of possible problems which can then arise, considering all combinations of "simple" boundary conditions which can exist on the remaining two edges, is 10. For a cylindrically curved panel (i.e. , a shell) the corresponding number is 136!

To complicate matters further, whereas all academicians will agree on the form of the classical, fourth order equations of motion for a plate, such agreement does not exist in shell theory. Numerous different shell theories have been derived and are used. Thus, if analytical results for frequencies and mode shapes of a given shell configuration are presented, strictly speaking, the shell theory used in the calculations must be specified. For the sake of separating and defining clearly the various shell theories commonly found in the shell vibration literature, chapter 1 is devoted to their derivation, with special emphasis being given to the identification of points in the derivation where the different assumptions are made which give rise to the different theories.

Statements are found in the literature which imply the equivalence of all eighth order shell theories. The accuracy of such statements is examined carefully in chapter 2 on a problem for which exact solutions exist--the closed circular cylindrical shell supported at both ends by shear diaphragms. Extensive comparisons of results from the various shell theories are made with those from the exact, three dimensional elasticity theory.

In addition to the differences in the theories, simplifications are often made in the resulting equations of motion or the characteristic (frequency) equations. These simplifications include, among others: neglecting certain (hopefully) small terms in the equations of motion, neglect of the tangential inertia terms, linearization of the characteristic equations, and assuming that the wave length in one direction is considerably longer than in the other. Comparisons of the effects of these simplifications are also made in chapter 2.

Comparing plate and shell vibrations, it is found that shell frequencies are more closely spaced and less easily identified, both analytically and experimentally. Furthermore, the fundamental (lowest frequency) made for a shell is generally not all obvious, whereas for a plate it usually is.

There are more parameters required to define the shell vibration problem. For example, consider a rectangular plate simply supported on all its edges. The complete frequency spectrum is determined by varying one parameter--the length-to width ratio. For the cylindrically curved panel having the same edge conditions, however, three *additional* parameters can be independently varied--the thickness-to-radius ratio, the length-to radius-ratio, and Poisson's ratio.

The present monograph contains approximately 1000 references. Of these, more than half deal with circular cylindrical shells. Therefore, two chapters were devoted to this voluminous topic. Chapter 2 deals with the results of classical theory while complicating effects are studied in chapter 3. By contrast, very little work has been done with noncircular cylindrical shells, and these results are summarized in chapter 4. Chapter 5 is devoted to circular conical shells.

Because of the complexity of the field of shell vibrations are described above, and because of my own limitations in time and organizational capability, the following sacrifices had to be made in the present monograph:

(1) There are undoubtedly more relevant references which have been unknowingly omitted from this work than in the previous one. This is mainly due to the comparative recentness of the shell vibrations literature.

(2) Chapter 6 is only a bibliography for the vibrations of spherical and other shells.

(3) Numerous forms of nondimensional frequency parameters as given in the literature are used directly without conversion to a common parameter.

For these shortcomings I sincerely apologize to all my readers.

The support of the National Aeronautics and Space Administration is gratefully acknowledged, particularly that of Mr. Douglas Michel, who sees the value of devoting time and effort to making available research results *useful* to mankind, as well as to the creation of new knowledge. I wish to thank Messrs. S.G. Sampath, Adel Kadi, and Ting-hwa Wang, three of my doctoral students, for their devotion to this work. Without their help in supervising the procurement of the relevant literature, in providing analytical help (particularly in chapters 1 and 2), and in editing the manuscript, this monograph would not have been possible--indeed, I would not have undertaken it. I also wish to thank Drs. Robert Fulton, Francis Niedenfuhr, Herbert Reismann, and Carl Popelar for their technical advice. Finally, the enormous editorial assistance of Mr. Chester Ball, and Mrs. Ada Simon is gratefully acknowledged.

Arthur W. Leissa
The Ohio State University