

**Crystal Acoustics**  
M.J.P. Musgrave  
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**Author's Preface**

"Crystal Acoustics" specifies the subject matter of this text as well as I can manage in two words and five syllables. I hope it is more self-explanatory than some designations offered but, if clarification be needed, the sub-title should fulfill this time-honored purpose.

Those who add to the literature of a well-established subject often have recourse to remarks of justification for "yet another book on...." The title I have chosen discharges me from that particular duty but, on the other hand, sets and even sterner task, namely, that of providing good reasons for adding an apparently new subject to the literature. In fact, the following chapters form an attempt to describe the conservative mechanics of waves in crystals whether regarded as continua or as structures of discrete particles. This, of course, is not a new subject, but I believe this is the first treatise which deals with both viewpoints in some detail.

The physical and chemical properties of crystalline solids have been of significance to mankind since the earliest times. Presumably, Stone Age man observed, and very probably studied, the phenomenon of cleavage-mechanical as well as anatomical! In primitive metalwork, the plastic properties of crystalline aggregates were certainly utilized. Genuine scientific inquiry is clearly the basis of Huygens' experiments with calcite, and his crystalline specimens were presumably obtained from travelers who were also amateur geologists. Many would acclaim Huygens, with good reason, as the founder of crystal optics. I would like to suggest that he could with equal, if not greater, justification be described as the founder of "crystal acoustics," for though his observations were made with light, his thinking was mechanical or

acoustical. Waves as then understood were conceptually similar to sound waves in air; longitudinal, acoustical.

As time passed, the theory of the propagation of light in crystals was formulated in terms of a special medium, the elastic solid ether whose remarkable properties allowed the transmission of waves of transverse, but not longitudinal, mechanical displacement. When, thanks to Maxwell, light came to be interpreted as an electromagnetic phenomenon, crystal optics and crystal acoustics became concerned with different physical concepts, though remaining unified by the mathematics of wave theory.

Since 1912, the study of crystals as elastically anisotropic continua has been complemented by Born's studies of crystal lattices which have developed into the modern theory of lattice dynamics. This theory is concerned to interpret the thermodynamic and optical properties of crystals in terms of the characteristic frequencies and displacements of the atomic vibrational modes. Although such effects as the absorption of light are interpreted as interactions of an incident electromagnetic wave with the dipole field within a crystal, a basic mechanics for the particles of the crystal is first necessary.

Accordingly, I have attempted a treatment of elastic waves propagating in anisotropic continua and in periodic structures of discrete particles with the object of drawing together the two well-developed strands of crystal mechanics. The concepts employed in such a discussion are, of course, idealized, but it is their relevance to crystal physics which is the principal concern of this book.

Consequently, it is a text intended to be of use to physicists, applied mechanicians, and electrical engineers who, for various reasons, study stress-wave propagation in crystals. In addition to providing detailed information about wave propagation, the various chapters of the book seek to provide an introduction to collateral subjects and to set up adequate signposts to further knowledge.

Much of Part I is also intended to be of interest to geophysicists, particularly seismologists, who are concerned with anisotropy in planetary structure. It is also highly relevant to the elastic properties of modern materials such as fiber composites. Chapter 1 contains a short historical sketch of past thinking and developments, while Chapter 2 sets the scene by emphasizing that structure in general implies anisotropy. Those impatient to reach the meat may begin at Chapter 3 which sets forth the basic ideas and definitions of elasticity theory. There follows Chapter 4 treating essential ideas of crystal symmetry and their implications for physical properties. Chapter 5 discusses some static deformations of crystals, under simple stress systems.

Chapters 6 and 7 treat the dynamics of an elastic solid, and the significance of characteristics in wave propagation and the properties of plane waves are developed. Chapters 8, 9, 10 treat in detail and illustrate the propagation surfaces of crystals of various symmetries. Chapter 11 deals with reflection and refraction and is followed by a discussion of surface waves (Chapter 12). Chapter 13 illustrates the use of Fourier transforms in elastic wave theory. Chapter 14 offers a brief discussion of stationary waves; Chapter 15 discusses the properties of polycrystalline aggregates; Chapter 16 deals briefly with some higher-order effects.

In Part II, Chapter 17 develops many essential features of lattice dynamics from a consideration of one-dimensional chains. Chapter 18 then presents a minimal amount of three-dimensional theory which is illustrated in Chapter 19, by the discussion of simple models of crystals. Chapter 20 indicates the limitations of harmonic theory and sketches the achievements of the theory of small anharmonicity.

It is hoped that the greater part of the book will be intelligible to readers who have completed first-degree syllabuses in mechanics and physics. Some knowledge of linear algebra and elementary calculus, together with geometrical awareness and a facility in the use of suffix notation, are the sole requirements for an appreciation of most of the text. Certain sections, notably in Chapters 13, 18 and 19 are more difficult; these have been marked in the summary of contents with an asterisk (\*)

Although one of the main objectives of the book is to juxtapose adequately referenced treatments of continuum and discrete particle models within the same cover, the two parts remain separable. Courses

based on either part alone, (though preferably both), may be designed as appropriate for a given audience.

An author's debts of gratitude are many and various and I would like to thank all those who have helped me to bring this book into being; in particular, my thanks go to Mr. F. H. Murphy and Mr. E. F. Riley for their patience over many months when I made little progress with the MS, and for the promptness with which they and their staff handled it when it reached their hands; to Miss M. Lewis for typing much different material; to Miss B. Mayer and Mrs. S.N. Kulpa who edited the MS; to Mr. Morland and his colleagues for the preparation of the diagrams; to Mr. H.L. Cox who read the MS and offered valuable comments; to Dr. T. C. Lim who also read the MS and offered not only shrewd comments but also some very recent results which are incorporated in Chapter 21 on surface waves; last, but not least, to my wife who helped in many different ways but especially in the compilation of the index.

For remanent errors I accept full responsibility, but I shall be grateful to any readers who care to bring them to my attention.

M. J. P. Musgrave  
London, 1970

### **Foreword to Reprint Edition**

On a memorable morning in August 2000, I was happily surprised to receive a letter from Mack Breazeale, my good friend ever since we first met in Stuttgart during the summer of 1959. Accompanying his letter was an enclosure which he urged me to consider very seriously; it was a formal proposal from the Acoustical Society of America to reprint *Crystal Acoustics* under its sponsorship.

Unhappily, my wife had been a permanent invalid for some years and, with an indifferent record of health personally, I had not been scientifically active for the better part of a decade. *Crystal Acoustics* had served as course text for postgraduate lectures for fifteen years prior to my retirement—a useful enough lifetime—but, during the 1990's, the need it hitherto fulfilled was seemingly on ebb.

Accordingly, I was delighted to learn that my monograph of 1970 had become a text, sufficiently widely read and cited, to merit reprint by a learned society of international standing.

My response to the offer was enthusiastic and positive.  
Here is the result.

Many errors of the original text have been corrected, but any which are yet remanent should be attributed to my oversight.

Until 1990, I published papers on topics in elasticity, and several of the more interesting separates, with relevance to *Crystal Acoustics*, have been conjoined within the covers of the reprint. Some may find this prefatory note superfluous, but I hope many more will welcome a brief review, over the thirty odd years since first publication, as apposite.

During the last three decades, the significance of anisotropy in the physical properties of condensed matter has been recognized by an increasingly diverse population within the scientific and engineering communities. No aspect of this diversity is more striking than the surge of geophysical interest reflected in a list of citations kindly prepared for me by Adrian Clarke, Mathematics Library, Imperial College; during 1981-1990, citations of *Crystal Acoustics* from geophysical titles numbered 32 out of 163 (19.6%); in 1991-2000, the count rose to 72 out of 292 (24.7%). Papers were theoretical or observational in roughly equal proportion.

There can be no doubt that the basic details of elastic wave propagation in anisotropic media have become far more widely appreciated by geophysicists. Let us not forget, however, that the transversely isotropic mineral, beryl, whose elastic stiffnesses were listed by Voigt<sup>1</sup>, has a long record as a simple model for the complex patterns of stratification in the earth's mantle<sup>2,3</sup>. Recent advances in the technology of seismometry have substantially diminished the gap between observational limits and theoretical models.

The years 1989-91 saw a spate of citations deriving from problems of anisotropy within geophysics; in particular, Sayers<sup>4</sup> discussed the inner core and Cheadle *et al.*<sup>5</sup> reported a model with orthorhombic symmetry.

This expansion of interest was confirmed and consolidated by the publication of Helbig's<sup>6</sup> book in 1994. The author takes every opportunity to geometrize his subject matter and supplements his text with a plentitude of helpful diagrams. Though prepared within the framework of a series of geophysical titles, most of the contents are of wider interest and germane to a more general readership.

Several more recent studies focusing on computational analysis of models and/or observational data include MacBeth *et al.*<sup>7</sup>, Rumpkin *et al.*<sup>8,10,12</sup>, Daley *et al.*<sup>9</sup>, Sileny *et al.*<sup>11</sup>.

Theoretical studies of sources in unbounded and bounded anisotropic media have been made by many authors. Most notable are the monograph by Payton<sup>13</sup> with detailed results for hexagonal media, and papers by Wang *et al.*<sup>14,15</sup>, Every<sup>16</sup>, Gajewski<sup>17</sup>, Gajewski *et al.*<sup>18</sup>, Payton<sup>19,20</sup>, Abrahams *et al.*<sup>21</sup> Related to calculated results there have been many experimental visualizations of elastic waves, especially of the complexities occurring in the neighbourhood of double, or conical, points on slowness and wave surfaces. See Every<sup>22,23</sup>, Every *et al.*<sup>24</sup>, Kim *et al.*<sup>25</sup>

Other experiments discussed in terms of phonon spectroscopy, reviewed by Wybourne *et al.*<sup>26</sup>, include measurements of elastic stiffnesses from Brillouin scattering performed by Tulk *et al.*<sup>27,28</sup>, Pang *et al.*<sup>29,30</sup>

Interesting universal relations and bounds for wave propagation parameters are discussed by Boulanger *et al.*<sup>31,32</sup>

Since 1970, a major advance in elastic problems, static and dynamic, involving plane surfaces or interfaces bounding anisotropic media, has been achieved by developing the work of Stroh<sup>33</sup>. Within the title *Crystal Acoustics*, some introduction to the considerable literature of the Stroh formalism as applied to surface and interface waves should surely be provided. SIAM Conference Proceedings, ed., Wu *et al.*<sup>34</sup>, fulfill such purpose effectively. More recently Chadwick<sup>35</sup> has applied the method to prestressed elastic media.

In parallel with this theoretical advance, but largely independent of it, surface acoustic waves have been widely applied in electronic devices, many of which include anisotropic components. In this connection, it is noteworthy that the treatise by Auld<sup>36</sup> includes details of computational procedures applicable to device components, in addition to comprehensive coverage of the theoretical and experimental aspects of acoustics in solids.

In a wider perspective, a commendable history of the design of surface acoustic wave (SAW) devices in electronic applications has been provided by Morgan<sup>37</sup>.

Whilst this retrospect does no more than draw attention to some of the significant developments of the last thirty years, I hope the preceding remarks evince that crystal acoustics as a branch of physics is alive and flourishing.

May this reprint help to maintain that vigour for another decade or two.

*Acknowledgements.* My grateful appreciation goes to all those who have helped to realize this reprint: in particular, advice and assistance in various ways have been courteously provided by Mack Breazeale, Elaine Moran, of the Acoustical Society of America; Adrian Clarke, Dan Moore, Mathematics Department, Imperial College, London; co-author and friend, Robert Payton; Andrew Gibbons, Royal Society; Oxford University Press.

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