Acoustics, Elasticity, and Thermodynamics of Porous Media
Twenty-One Papers by M. A. Biot
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Paper 19. Variational-Lagrangian Irreversible Thermodynamics of Initially-Stressed Solids with Thermomolecular Diffusion and Chemical Reactions. *Journal of the Mechanics and Physics of Solids* **25**, 289-307 (1977); 26, 59 (Errata) (1978)

Paper 20. Variational Irreversible Thermodynamics of Heat and Mass Transfer In Porous Solids: New Concepts and Methods. *Quarterly of Applied Mathematics* **36**, 19-38 (1978)

Paper 21. Variational Irreversible Thermodynamics of Physical-Chemical Solids with Finite Deformation. *International Journal of Solids Structures* **14**, 881-903 (1978)

Introduction

Biot's theory of the mechanics of porous media is remarkable on several counts. It is a major twentieth century extension of theoretical continuum mechanics—a generalization of elasticity theory to realistic materials with liquid-filled pore spaces which, in its final form, incorporates a complete spectrum of thermodynamical and dissipative effects. It has led to the solution of numerous problems of soil consolidation, dynamics and wave propagation in acoustics, geophysics, engineering and applied physics—problems beyond the scope of traditional methods of elasticity theory and for which the Biot theory constituted an important breakthrough. Finally, and quite remarkably, it was created almost exclusively by one man: M.A. Biot (1905-1985).

Twenty-one papers have been collected in this volume. Originally selected by Biot as the basis for a monograph, they were to have been woven into a whole by a connecting text and appendices, which would help the reader cope with aspects of the treatment which, over the years, the author had taken for granted but which might not always seem transparent for the unprepared. This material, alas, was never written—although most, and possibly all of it, can be extracted from Biot's other books and monographs.1-3 The papers are given here in the sequence intended by their author, i.e., in the historical order of their appearance over a period of almost forty years. They have been accordingly numbered **Papers 1-21**(see Table of Contents). The only alteration to this material has been the correction of a number of misprints.

Paper 1,(1941), establishes the fundamental field equations for three-dimensional consolidation of an *isotropic* model representing the settlement of soil under a load. These equations give the stresses and displacements of an elastic matrix, or skeleton, whose voids are filled with a viscous fluid satisfying Darcy's law. Whereas the basic theory is simple and straightforward, its implications are considerable, as it establishes the conceptual framework from which stem the generalizations of Biot's later work. Assuming isotropy, linearity, small strains, reversibility and an incompressible fluid, a system of four linear partial differential equations is obtained for the four unknowns *u,v,w,o* (displacements of the solid matrix and the fluid pressure). These equations are second-order in *x,y,z* and first order in *t* and allow one to solve the time-dependent diffusion-type settlement problem for concentrated loads and linear boundary conditions. The properties of this system are determined by four distinct physical constants. The paper also demonstrates how the example of a suddenly applied load can be solved in a few lines using elementary Heaviside operational calculus (as set forth, for instance, in Kármán and Biot's textbook1).

Papers 2, 3 and 4 are illustrative: they show how the methods established in **Paper 1** are applied to specific settlement/consolidation problems.

Paper 5 deals with the more general case of the *anisotropic* solid, the pore spaces of which are filled with a *compressible* viscous fluid. This model requires 28 distinct elastic coefficients plus 6 for the generalized Darcy flow. In the common case of transverse isotropy these reduce to 8+2 constants.

Paper 6 returns to the isotropic case showing that it is possible to derive general solutions similar to the Boussinesq-Papkovich results of the theory of elasticity.

Papers 7 and 8 introduce the forces of inertia into the elastic stress-strain relations of the consolidation models of **Papers 1-6** to give a dynamical small motion theory—i.e., the equations of wave propagation in a porous fluid-saturated solid. These articles embody the fundamentals of what is now referred to as *the Biot theory* of porous media. Intensive research correlating this theory with laboratory and field measurements is reported in the acoustical and geophysical literature of the late 1970's and early 1980's.[4,5,6] One of the theory's interesting prediction was the possibility of a strongly attenuated *slow acoustic mode*: its existence was reported experimentally in 1980 by T.J. Plona[7]—twenty-four years later. A thorough overview of the field's status with particular emphasis on the theory's ability to accurately predict the attenuation properties of sediments and suspensions, is found in a 1985 series of articles by P.R. Ogushwitz. Papers 7 and 8 have been seminal for understanding the propagation and attenuation of sound and shear modes in sediments and rock, although widespread recognition of their importance in the literature took about twenty year. Today they are classics and remain among the most widely quoted papers in the field.

Paper 9 extends the theory to anisotropic solids with viscoelastic properties. This important paper ties in the purely poroelastic theory of Papers 1-8 with Biot's work on irreversible processes[8,9], introducing the Lagrangian methods and operational techniques which were to be the hallmarks of most of his later generalizations. The reader wishing to become acquainted with Biot's use of these is referred to his 1970 monograph *Variational Principles in Heat Transfer*.[3]

Paper 10, written with D.G. Willis, gives a discussion of the phenomenological significance and measurement methods of the elastic coefficients of the Biot theory, for both the strictly linear case *and* for the incremental coefficients in prestressed systems.

Paper 11. Here Biot pursues the generalization of the theory, further linking the propagation of waves in anisotropic, viscoelastic media with thermodynamical principles. He employs herein a *correspondence principle* which states that the equations governing the mechanics of porous media are the same for elastic and viscoelastic models providing that, in the latter case, the elastic coefficients are replaced by the corresponding operators—a principle similar to that used for *homogeneous* viscoelastic (nonporous) rock (see e.g. Jeffreys[10]). The concept of viscoelasticity used here is quite general, in that it encompasses all relaxation mechanisms, whatever their nature—i.e. thermoelastic, electric, physico-chemical, or purely mechanical processes. In the limit of vanishing fluid density these equations yield the theory of thermoelastic propagation in nonporous materials—a direct consequence of the isomorphism between the theories of porous media and thermoelasticity demonstrated elsewhere.[9]

Page 12 applies the general theory outlines in Paper 11 in the specific context of elastic wave propagation in dissipative porous media, i.e., the cases of acoustic and seismic waves in realistic earth and rock models. Like Paper 11, this paper underlines the generality and power of the theory, which can be made to accommodate a great variety of dissipative mechanisms.

Paper 13 represents the conjunction of Biot's theory of porous media with another, previously independent, strand of his research: the mechanics of elastic media under initial stress, which had been formulated in a series of papers between 1935 and 1964 and is the subject of his classic 1965 monograph *The Mechanics of Incremental Deformations*[2] (which dealt not only with the relevant wave equations but also with the geophysically important problems of stability in layered viscoelastic media, thus offering the earth sciences the first systematic and physically correct theory of the initial stages of geological folding).

Following a brief engineering application of the theory in **Page 14** (buckling of a porous slab), Biot returns in **Papers 15 and 16** to the difficult domain of finite strain, applying his unique combination of incremental mechanics, operational and Lagrangian techniques to describe a broad class of nonlinear effects in porous media—results having a range of applications in engineering, acoustics and geophysics.

Paper 17 is an intriguing aside showing how it is possible, for some types of symmetry, to apply the hodograph method of classical mechanics to problems of finite stress analysis for porous-walled structures (cylinders and spheres).

Papers 18-21 give the final and most general form of Biot's theory of porous media—as an offshoot of his formulation of irreversible thermodynamics, using a principle of virtual dissipation, Lagrangian methods and new concepts such as thermobaric and convective potentials. This leads to flexible new techniques for dealing with a remarkable breadth of phenomena. Together with another review article,[11] these three papers represent the culmination of Biot's work in these fields. They contain the seeds of much future research, materials for theses and new insights into acoustics, applied mathematics, the earth, and engineering sciences.

This collection offers a coherent, systematic treatment not only of the widely used Biot theory of porous media (**Papers 1-11**), but also of the more general theory developed later (**Papers 12-21**). It took approximately twenty years for the material in **Papers 1-11** to become assimilated by acousticians and geophysicists, who will now be able to profit from this volume to anchor and deepen their understanding of the theoretical mechanics of porous media. It may be hoped that **Papers 12-21** will hasten our assimilation of the more general and powerful theory developed by Biot in the years 1962-1978. In an era dominated by computer-oriented research and the sometimes indiscriminate use of vast numerical programs for the solution of complex problems, physical insight of the kind characterizing Biot's work is rare—yet it is probably a *sine quo non* for basic progress in our understanding of the behavior of real materials in acoustics, the earth sciences and engineering science.

This set of 21 articles offers an unusual document setting forth the creation, in modern times, by one man, of a new classical theory in continuum mechanics—from deceptively simple beginnings as a quasi-static, isotropic model of consolidation in **Paper 1** to, in **Paper 21**, a sophisticated general theory of anisotropic, viscoelastic, porous dissipative media taking into account a full spectrum of thermoelastic, physico-chemical and thermodynamic phenomena.

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